

Local Energy Velocity of Classical Fields

Language to be improved

I. V. DROZDOV *

and

A. A. Stahlhofen †

University of Koblenz, Institut für Naturwissenschaften

Abteilung Physik

Universitätsstr.1, D-56080 Koblenz, Germany

February 1, 2008

Abstract

It is proposed to apply a recently developed concept of local wave velocities to the dynamical field characteristics, especially for the canonical field energy density. It is shown that local energy velocities can be derived from the lagrangian directly. The local velocities of zero- and first- order for energy propagation has been obtained for special cases of scalar and vector fields. Some important special cases of these results are discussed.

1 Introduction

The terms of "propagation" and "velocity" supposed to characterize a wave propagation can in case of classical fields be treated as a propagation of a local attribute, as shown recently This approach provides a spectrum of local phase velocities.

These local velocities, being itself local dynamical fields, stage an alternative representation of the original physical field and their behaviour under several important local symmetries is the subject of investigation for physical applications. For instance, the two- and three- dimensional generalizations of zero- and first-order velocities to co-variant and contra-variant vectors respectively are established [2]

*e-mail: drosdow@uni-koblenz.de

†e-mail: alfons@uni-koblenz.de

Especially, it has been shown [1], that the zero-order velocity is Lorentz covariant in sense of the relativistic addition of velocities. This property preserves the local relativistic causality on the zero-order level under change of frame. For local velocities of higher orders the local Lorentz covariance does not hold and the subluminal restriction in arbitrary frame is not assured. Moreover, simplest counter-examples have been demonstrated [1, 3] where the first order propagation velocity may be not restricted by c for the global propagation between two points.

In the framework of discussion on a relativistic causality and its violation by several local velocities overcoming the fundamental constant c , the following remarks are in order.

The relativistic causality still remains kept, as long as the attribute propagating is only measured in two different space-time points and the propagation means no real transmission of "some physical feature" under consideration from the one point to the other one. Superluminal phase and group velocities, for instance, are well known.

Analogous, the propagation velocity of zero order, i.e. propagation of the attribute itself, may approach unbounded values overcoming the light velocity, without to controverse the relativistic causality immediately.

On the contrary, one should proceed carefully with velocities of higher order. Since the propagation of the attribute like a derivative can be treated as one of a state alternation in a physical system (i.e. by an interaction process), the propagation might be interpreted as an interaction transmission or information transmission respectively. Therefore, this velocity may concern with a real physical phenomenon and deserves a consistent interpretation.

2 Local energy velocity, force velocity etc.

The subject of "energy velocity" being dealt with in the optics, is usually interpreted in context of continuity equation containing only the global energy density of the all matter in the space and its current flow density respectively. An assignment of any velocity to this current for an arbitrary configured field is therefore an ambiguous non-local procedure.

Instead of that, the energy state of a certain field of the field system can be observed, whereat the corresponding energy density T^{00} (zero component of Noether's conserved current) can be considered as a scalar field. Therefore, the formalism of local velocities [1] can be applied here.

Basically, for any scalar field $\varphi(x, t)$ the velocity for a corresponding local

attribute propagation is defined as the N-th order phase velocity (N-PV):

$$v_{(0)}(x, t) := \frac{dx^f}{dt} = -\frac{\dot{\Psi}}{\Psi'} \quad \text{0-PV (field velocity)} \quad (2.1)$$

$$v_{(I)}(x, t) := \frac{dx^p}{dt} = -\frac{\dot{\Psi}'}{\Psi''} \quad \text{1-PV (peak velocity)} \quad (2.2)$$

$$v_{(II)}(x, t) := \frac{dx^{TP}}{dt} = -\frac{\dot{\Psi}''}{\Psi'''} \quad \text{2-PV (turning point velocity)} \quad (2.3)$$

and so on [1].

In the approach of the field theory, the state of a physical object is described by the field $\Psi(x, t)$, treated, for instance, as a density of probability distributed in space-time.

Let us start the discussion by briefly recalling the local concept. The observation of a time evolution of the field state in accordance with the equation of motion provides for a fixed field value Ψ_0 the propagation velocity $v_{(0)}$ of zero order (0-PV).

A certain field state is described by the corresponding distribution of energy density T^{00} ; the fixed value \mathcal{E} of it propagates with the 0-PV $w_{(0)}$. In this sense it deals entirely with an alternative -in this case energetic- representation of the field Ψ by the field \mathcal{E} . Nevertheless it does not mean an equivalence of both approaches, especially of two propagation velocities $v_{(0)}$ and $w_{(0)}$.

In fact, the possible discrepancy is due to the structure of lagrangian for the field Ψ and the velocities $v_{(0)}$ and $w_{(0)}$ are in general never equivalent. This fact affords no contradictions to basical statements of physics, e.g. energy conservation, since these velocities are being interpreted properly as local features. Namely, an alternation of the field Ψ in the point x_0 at the time t_0 implies an alternation of the corresponding energy density. In the next moment t_1 the same field value Ψ will be recovered in a neighbouring point x_1 because of the field "re-configuration" due to the natural field dynamics (equation of motion). Since the variation of the energy state involves also an energy for this re-configuration, the point x_2 where the initial local energy density is to recover, will not be the same as x_1 . In other words, the energy density includes a kinetic contribution of the dynamical re-shaping. The zero-order energy velocity should involve therefor at least the first order field velocity together with the zero-order one.

The question, how local energy velocities are related to the local field velocities, is discussed in following sections of the present work. A formal analysis of the lagrangian procedure is associated with the form and special properties of lagrangian (e.g. symmetries) and produces no general predictions.

For instance, for the lagrangian density under consideration being an arbitrary function of the field Ψ and its first derivatives, so that

$$\mathcal{L} = \mathcal{L}(\Psi, \dot{\Psi}, \Psi', \Psi_k), \quad (2.4)$$

(where the dot denotes the derivative with respect to time, the prime - with respect to the chosen direction x , and the index k - with respect to other directions

$\{y, z\}$), the energy density of Ψ reads [4]

$$\mathcal{E} = T^{00} = \dot{\Psi} \frac{\partial \mathcal{L}}{\partial \dot{\Psi}} - \mathcal{L} \quad (2.5)$$

and one obtains for the zero order energy velocity

$$w_{(0)} = - \frac{\frac{\partial^2 \mathcal{L}}{\partial \dot{\Psi} \partial \Psi} \dot{\Psi}^2 + \frac{\partial^2 \mathcal{L}}{\partial \dot{\Psi}^2} \ddot{\Psi} \dot{\Psi} + \frac{\partial^2 \mathcal{L}}{\partial \dot{\Psi} \partial \Psi_k} \dot{\Psi}_k \dot{\Psi} + \frac{\partial^2 \mathcal{L}}{\partial \dot{\Psi} \partial \Psi'} \dot{\Psi}' \dot{\Psi} - \frac{\partial \mathcal{L}}{\partial \Psi} \dot{\Psi} - \frac{\partial \mathcal{L}}{\partial \Psi_k} \dot{\Psi}_k - \frac{\partial \mathcal{L}}{\partial \Psi'} \dot{\Psi}'}{\frac{\partial^2 \mathcal{L}}{\partial \dot{\Psi} \partial \Psi} \dot{\Psi} \Psi' + \frac{\partial^2 \mathcal{L}}{\partial \dot{\Psi}^2} \dot{\Psi} \ddot{\Psi}' + \frac{\partial^2 \mathcal{L}}{\partial \dot{\Psi} \partial \Psi_k} \dot{\Psi} \Psi'_k + \frac{\partial^2 \mathcal{L}}{\partial \dot{\Psi} \partial \Psi'} \dot{\Psi} \Psi'' - \frac{\partial \mathcal{L}}{\partial \Psi} \Psi' - \frac{\partial \mathcal{L}}{\partial \Psi_k} \Psi'_k - \frac{\partial \mathcal{L}}{\partial \Psi'} \Psi''} \quad (2.6)$$

Especially, in view of a discussion on the relativistic causality, is it sufficient to assure whether the local energy velocity could not overcome the field velocity itself or moreover exceed the light velocity in vacuum.

In terms introduced above the condition of relativistic causality of energy for zero order can be formulated e.g. as follows:

$$\dot{\Psi}^2 \partial_m \left[\frac{\partial \mathcal{L}}{\partial \dot{\Psi} \partial \Psi_m} \right] \leq \dot{\Psi} \left(\frac{\partial \mathcal{L}}{\partial \dot{\Psi}} \right)' - \frac{\partial \mathcal{L}}{\partial \Psi} \Psi' - \frac{\partial \mathcal{L}}{\partial \Psi_k} \Psi'_k - \frac{\partial \mathcal{L}}{\partial \Psi'} \Psi'' \quad (2.7)$$

On the one hand, several physical requirements like these mentioned above arrange for an interpretation of energy velocity for a given lagrangian.

On the other hand, they impose certain restriction on the construction of lagrangians describing an admissible "non-exotic" matter.

We proceed with local velocities of higher order for energy density in order to give an interpretation of the subject of interaction velocity, as outlined in the introduction.

Suppose, the field Ψ expires at the moment t_0 a small perturbation: for example, the value Ψ_0 changes in the point x_0 stepwise to the Ψ_1 . According to the equation of motion ("natural dynamics") this steplike change will be observed in the point x_1 at the moment t_1 . A pure distributive field shape is not realisable physically and looks in fact as a kink. The velocity of the propagating kink can be represented by the turning point velocity 2-PV. The perturbation of the field means also the alternation of energy density in this point and leads to the result, the energy density in the point x_1 at later moment will change as well. A simplest signal of changed shape is for instance the first derivative of the shape. In this sense, the 1-PV of energy density can be interpreted as a propagation of interaction. Here is it not necessary to indentify this phenomena with an "energy transmission" or a "transmission" at all. The energy density descends local in one point and as a result it ascends in another one, as well as a field value Ψ . The subject of "transport" is irrelevant for this phenomena, since these values are not labelled to identify. In other words, one cannot say, the energy of field density ascending in the point x_1 is "the same" one leaving the point x_0 . The energy density of a single field Ψ is also not subjected to the conservation in form of some continuity equation, since the lagrangian may involve any interactions with other

fields, but only the total energy of the system of fields remains conserved. The formal analysis, discussed above, will be demonstrated now on simplest and well known field-theoretical models.

2.1 Scalar field

For a 1-dimensional field shape $\varphi = \varphi(x, t)$ of a massive scalar field with the lagrangian density

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2 - V(\varphi) = \frac{1}{2}\{\dot{\varphi}^2 - \varphi'^2 - m^2\varphi^2\} - V(\varphi), \quad (2.8)$$

($c = \hbar = 1$) the corresponding energy density reads:

$$\mathcal{E} = T^{00}(x, t) = \frac{1}{2}\{\dot{\varphi}^2 + \varphi'^2 + m^2\varphi^2\} + V(\varphi). \quad (2.9)$$

The zero order energy propagation velocity w_0 is defined as

$$w_{(0)} = -\dot{\mathcal{E}}/\mathcal{E}'. \quad (2.10)$$

Additionally, it holds for the field φ the equation of motion (Klein-Gordon wave equation)

$$\ddot{\varphi} + m^2\varphi + \frac{\partial V}{\partial \varphi} = \varphi''. \quad (2.11)$$

This equation , derived once again with respect to x provides the relation:

$$\ddot{\varphi}' + m^2\varphi' + \frac{\partial^2 V}{\partial \varphi^2}\varphi' = \varphi'''. \quad (2.12)$$

Now, a direct application of the definition (2.10) of zero-order phase velocity (0-PV) to the energy density $T^{00}(x, t)$ gives:

$$w_{(0)} = -\frac{v_{(0)} + v_{(I)}}{1 + v_{(0)}v_{(I)} + \frac{1}{\varphi''}\left[m^2\varphi + \frac{\partial V}{\partial \varphi}\right]} = \frac{v_{(0)} + v_{(I)} + 2m^2\varphi}{v_{(0)}v_{(I)} + \frac{\ddot{\varphi}}{\varphi''}}, \quad (2.13)$$

where $v_{(0)}, v_{(I)}$ are the 0- and 1-PV's of the field shape respectively. The equation of motion (2.11) is in this relation encountered.

For massless fields with a constant potential it follows, the zero-order energy velocity is a relativistic addition of zero-(field) and first order (peak) velocities of the field.

Proceeding in a similar way with the first order energy velocity and using the relations (2.11, 2.12), one obtains:

$$w_{(I)} = \frac{v_{(0)} + v_{(II)} + 2Cv_{(I)}}{Cv_{(I)}^2 + v_{(0)}v_{(II)} + \frac{C}{\varphi''}\left[m^2\varphi + \frac{\partial V}{\partial \varphi} + \varphi''\right] + \frac{1}{\varphi'''}\left[m^2\varphi' + \frac{\partial^2 V}{\partial \varphi^2}\varphi' + \varphi'''\right]} \quad (2.14)$$

with the notation

$$C := \frac{\varphi''^2}{\varphi''' \varphi'} \quad (2.15)$$

for the dimensionsless factor C , dependent on the "shape curvature" of the field. It follows from the relation of $w_{(I)}$, for the turning point (TP) as a field shape attribute being traced ($\varphi'' = 0$) the curvature factor disappears and the expression becomes similar to that for the $w_{(0)}$:

$$w_{(I)}^{TP} = \frac{v_{(0)} + v_{(II)}}{v_{(0)}v_{(II)} + \frac{1}{\varphi''} \left[m^2 \varphi' + \frac{\partial^2 V}{\partial \varphi^2} \varphi' + \varphi''' \right]} \quad (2.16)$$

Hence we can see again, for a massless scalar field φ with the potential V linear with respect to the field, the first order local velocity of the perturbation of the energy density is a relativistic addition of zero-(field velocity) and second order (turning point) velocities. This velocity can be interpreted as a propagation of a step-like (or "discontinuity point") singal in the scalar field.

2.2 Massless vector field (electrodynamics)

Starting with the Fermi lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2, \quad \text{with} \quad F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \quad (2.17)$$

one obtains the source-free Maxwell equation

$$\partial_\nu F^{\mu\nu} = 0 \quad (2.18)$$

and the energy-momentum tensor [5]

$$T_\nu^\mu = \left(F^{\mu\rho} F_{\rho\nu} - \frac{1}{4} \delta_\nu^\mu F^2 \right), \quad (2.19)$$

that means for the energy density in terms of usual physical fields:

$$\mathcal{E} = T_0^0(x, t) = \frac{1}{2}(\mathbf{E}^2 + \mathbf{H}^2). \quad (2.20)$$

For a source-free electromagnetic wave obeying the condition $\mathbf{E} \perp \mathbf{H}$, we can choose a special cartesian coordinate system $\{x, y, z\}$, such that

$$\mathbf{E} = (E, 0, 0), \quad \mathbf{H} = (0, H, 0). \quad (2.21)$$

Hence, the zero-order energy velocity in the z -direction reads:

$$w_0 = -\frac{E\dot{E} + H\dot{H}}{EE' + HH'} \quad (2.22)$$

where the dot and the prime denote the derivative with respect to time t and direction z . respectively. Using the Maxwell equations :

$$\dot{H} = c \frac{\partial}{\partial z} E = -cE'; \quad \dot{E} = -c \frac{\partial}{\partial z} H = -cH'; \quad \frac{\partial}{\partial y} E = c \frac{\partial}{\partial x} H = 0; \quad (2.23)$$

we obtain, that

$$w_0 = c \frac{EH' + HE'}{EE' + HH'}. \quad (2.24)$$

Moreover, since the electric and magnetic field components are lineary connected through the "wave impedance" Z by

$$E = ZH, \quad (2.25)$$

the zero order energy velocity for transversal electromagnetic field propagation is constant:

$$w_0 = c \frac{Z}{Z^2 + 1} \quad (2.26)$$

and for each real value Z of wave resistance it cannot overcome the light velocity. The maximal absolute value $c/2$ can be reached at $Z = \pm 1$.

Performing the first order (1-PV) energy velocity in a similar way, one obtains

$$w_I = - \frac{E'\dot{E} + H'\dot{H} + E\dot{E}' + H\dot{H}'}{EE'' + HH'' + E'^2 + H'^2} = c \frac{2Z}{Z^2 + 1}, \quad (2.27)$$

with a maximal value c . It can be interpreted in a general meaning as a propagation of a spatial derivative of energy density (e.g. a peak of this) and can be called for this reason "force velocity". This object can be treated therefore physically in context of interaction transmission or signal transmission.

An application of this concept to the electrodynamics in media, which could have been of a greatest interest in this context, in view of possible application in photonics and near-field optics, is actually restricted through the lack of consistent lagrangian formalism in this theory, in the pure field-theoretical sense.

3 Conclusions

The concept of local velocities, applicable for arbitrary fielda, can be applied also for the energy density as a dynamcal field as well. It implies certain relations between local energy and field velocities. These relations are in general non-trivial and depends on the construction of the largangian density. For "normal" scalar and free electro-magnatic fields with conventional lagrangians this formalism leads also to non-trivial results, which are however apparently meaningful and physically non-controversal, and allow a consistent interpretation. It has been argued, for instance to treate the first order energy velocity as a speed of interaction in the field. This possibility is of course not unique, and the discussion is still open.

References

- [1] I. Drozdov, A. Stahlhofen, arxiv/ math-ph /0612023 and math-ph /0704.1485, submitted to Phys.Lett.A
- [2] I. Drozdov, A. Stahlhofen, *arXiv: math-ph/0701022, to be published*
- [3] I. Drozdov, A. Stahlhofen, *Talk on DPG-Workshop*
Quant.Opt.& Photon.Q III 23.03/07 Düsseldorf,
paper in preparation.
- [4] L. H. Ryder *Quantum Field Theory* Cambridge Univ.Press, 1985
- [5] L. D. Landau, E. M. Lifshitz, *Theoretical Physics, Vol.II Field Theory* ,
"Nauka" Moscow 1986